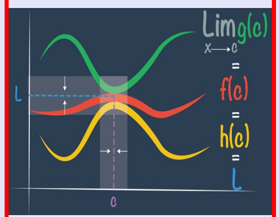
Calculus I

Lecture 29



Feb 19-8:47 AM

Given $f(x) = 2x^2 - x^3$

$2x^2 - x^3 = 0$
$x^2(2-x) = 0$
$x=0 \quad x=2$

1) Graph $f(x)$

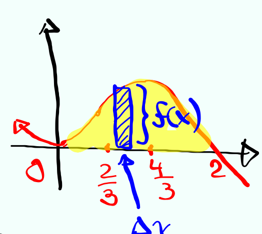
Domain $(-\infty, \infty)$

Y-Int $(0,0)$ X-Ints $(0,0), (2,0)$

$f'(x) = 4x - 3x^2 \to x(4-3x)$ C.P. $(0, \quad), (\frac{4}{3}, \quad)$

$f''(x) = 4 - 6x$ P.I.P. $(\frac{2}{3}, \quad)$

x	$-\infty$ to 0	0 to $\frac{2}{3}$	$\frac{2}{3}$ to $\frac{4}{3}$	$\frac{4}{3}$ to ∞
f'	−	+	+	−
f''	+	+	−	−
f	↘	↗	↗	↘

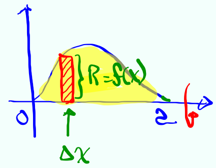


2) Find the area bounded in QI.

$$A = \int_0^2 f(x)\,dx = \int_0^2 (2x^2 - x^3)\,dx = \boxed{}$$

Jun 4-8:53 AM

3) Rotate the region bounded in QI by x-axis, find its volume.

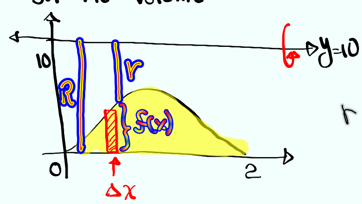


Disk

$$V=\int_0^2 \pi R^2\,dx$$

$$=\pi\int_0^2 \left[2x^2-x^3\right]^2 dx = \boxed{}$$

4) Rotate that region by $y=10$. Set-up the integral for the Volume



washer

$R = 10$

$r = 10-(2x^2-x^3)$

$r + f(x) = 10$

$r = 10 - f(x)$

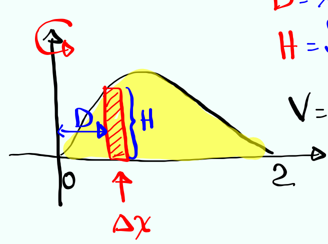
$= 10-(2x^2-x^3)$

$$V=\int_0^2 \pi\left[R^2 - r^2\right]dx$$

$$=\pi\int_0^2 \left[10^2-(10-2x^2+x^3)^2\right]dx$$

Jun 4-9:04 AM

Rotate that region by y-axis. Find the Volume



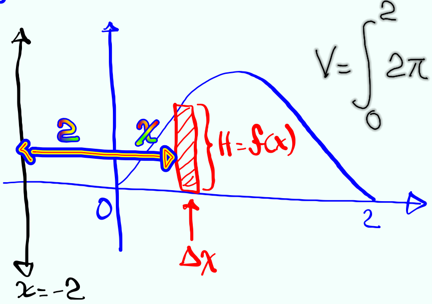
Shell

$D = x$

$H = f(x)$

$$V=\int_0^2 2\pi\, x(2x^2-x^3)\,dx = \boxed{}$$

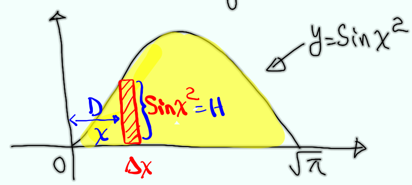
If we rotate about $x=-2$,



$$V=\int_0^2 2\pi\underbrace{(2+x)}_{D}\cdot\underbrace{(2x^2-x^3)}_{H}dx$$

Jun 4-9:12 AM

Consider the region below



1) Set-up the integral for the area.

$$A=\int_0^{\sqrt{\pi}} \sin x^2\,dx$$

2) Find the volume if it is rotated by Y-axis.

Shell $\quad D=x \quad H=\sin x^2$

$$V=\int_0^{\sqrt{\pi}} 2\pi\, x \sin x^2\, dx$$

$u=x^2$

$du=2x\,dx$

$x=0 \quad u=0^2=0$

$x=\sqrt{\pi} \quad u=(\sqrt{\pi})^2=\pi$

$$=\int_0^{\pi} \pi \sin u\, du$$

$$=\pi\cdot -\cos u\Big|_0^{\pi}=-\pi\left[\cos\pi-\cos 0\right]$$

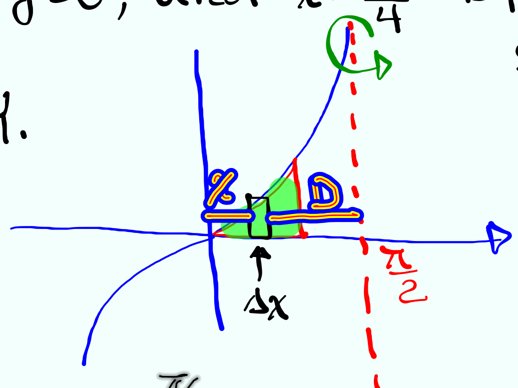
$$=-\pi(-2)=\boxed{2\pi}$$

Jun 4-9:19 AM

Rotate the region bounded by

$y=\tan x$, $y=0$, and $x=\frac{\pi}{4}$ by $x=\frac{\pi}{2}$.

Set-up only.



Shell

$D=\frac{\pi}{2}-x$

$H=\tan x$

$x+D=\frac{\pi}{2}$

$D=\frac{\pi}{2}-x$

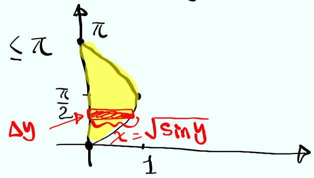
$$V=\int_0^{\pi/4} 2\pi\left(\frac{\pi}{2}-x\right)\tan x\, dx$$

Jun 4-9:27 AM

Consider the region bounded by

$x = \sqrt{\sin y}$, $0 \le y \le \pi$

1) Draw it



2) Set-up integral for the area.

$$A = \int_0^{\pi} \sqrt{\sin y}\, dy$$

3) Set up integral for volume if rotated about

a) $y = 4$

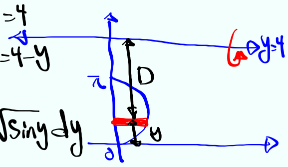
shell

$D = 4 - y$

$L = x = \sqrt{\sin y}$

$D + y = 4$

$D = 4 - y$



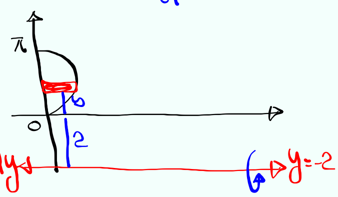
$$V = \int_0^{\pi} 2\pi (4-y)\sqrt{\sin y}\, dy$$

b) $y = -2$

Shell

$D = y + 2$

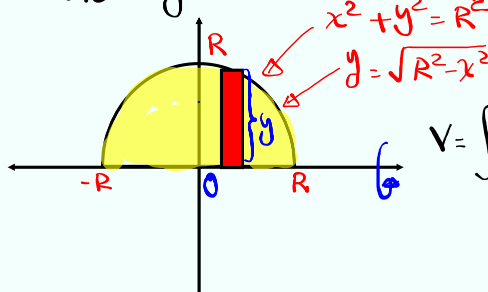
$L = \sqrt{\sin y}$



$$V = \int_0^{\pi} 2\pi (y+2)\sqrt{\sin y}\, dy$$

Jun 4-9:33 AM

Take the top half of the circle $x^2 + y^2 = R^2$, rotate by x-axis. Find its volume.



Disk

$$V = \int_0^R 2\pi \left[\sqrt{R^2 - x^2}\right]^2 dx$$

symmetry

$$= 2\pi \int_0^R (R^2 - x^2)\, dx = 2\pi \left[R^2 x - \frac{x^3}{3}\right]\Big|_0^R = 2\pi\left[R^2 R - \frac{R^3}{3} - 0\right]$$

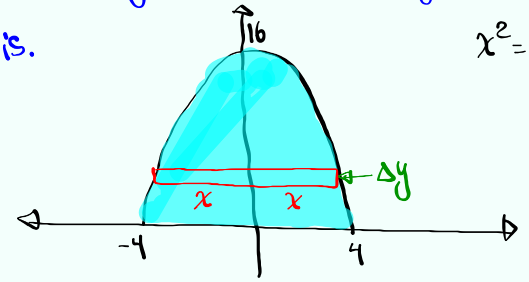
$$= 2\pi\left[R^3 - \frac{1}{3}R^3\right] = 2\pi \cdot \frac{2}{3} R^3 = \boxed{\frac{4\pi R^3}{3}}$$

Volume of Sphere

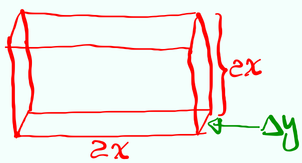
Jun 4-9:44 AM

Draw the region bounded by $y = 16 - x^2$ and x-axis.

$x^2 = 16 - y$



Cross-Sections in the form of square $\perp$ Y-axis.



$2x \cdot 2x \cdot \Delta y$

$V = \int_0^{16} 4x^2 \, dy$

$= 4 \int_0^{16} (16 - y) \, dy$

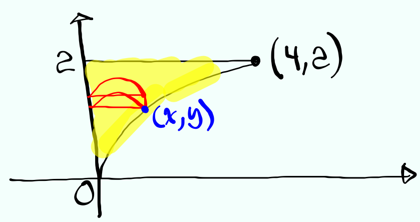
$= \boxed{}$

Jun 4-9:53 AM

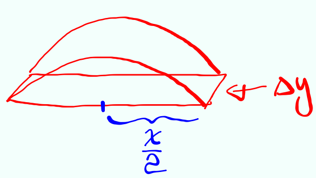
Consider the region bounded by $x = 0$, $y = 2$, and $y = \sqrt{x}$.

$y^2 = x$

$y^4 = x^2$



Cross-Sections in the form of Semi Circle $\perp$ Y-axis.



$V = \frac{\pi R^2 h}{2}$

$V = \frac{\pi}{2}\left(\frac{x}{2}\right)^2 \Delta y$

$= \frac{\pi}{8} x^2 \Delta y$

$V = \int_0^2 \frac{\pi}{8} x^2 \, dy = \frac{\pi}{8} \int_0^2 y^4 \, dy = \frac{\pi}{8} \cdot \frac{y^5}{5} \Big|_0^2 = \frac{32\pi}{40}$

$= \boxed{\frac{4\pi}{5}}$

Jun 4-10:01 AM

Work Problem

work required to move an object from $x=a$ to $x=b$ is given by

$$W=\int_a^b f(x)\,dx$$

where $f(x)$ is a force applied to the object to get moved.

Suppose an object is x units from the origin and a force of x^2+2x is being applied to the objects.

How much work is required to move the object from 1 to 2?

$$W=\int_1^2 (x^2+2x)dx=\left(\frac{x^3}{3}+x^2\right)\Big|_1^2=\left(\frac{2^3}{3}+2^2\right)-\left(\frac{1^3}{3}+1^2\right)$$

$$=\frac{8}{3}+4-\frac{1}{3}-1$$

$$=\frac{7}{3}+3=\frac{16}{3}$$

Units.

Jun 4-10:11 AM

Find f_{ave} for $f(x)=\dfrac{3}{(1+x)^2}$ over $[1,6]$

$$f_{ave}=\frac{1}{b-a}\int_a^b f(x)\,dx$$

$$=\frac{1}{6-1}\int_1^6 \frac{3}{(1+x)^2}dx$$

$u=1+x$

$du=dx$

$$=\frac{1}{5}\int_2^7 \frac{3}{u^2}du$$

$$=\frac{3}{5}\int_2^7 u^{-2}du=\frac{3}{5}\,\frac{u^{-1}}{-1}\Big|_2^7$$

$$=\frac{-3}{5}\left[\frac{1}{u}\Big|_2^7\right]=\frac{-3}{5}\left(\frac{1}{7}-\frac{1}{2}\right)=\frac{-3}{\cancel{5}}\cdot\frac{\cancel{-5}}{14}$$

$$=\boxed{\frac{3}{14}}$$

Jun 4-10:20 AM